Spherical Harmonic Representation of the Gravity Field Potential

1 Introduction

Satellites in low-Earth orbit are affected by a broad spectrum of perturbations due to the Earth's gravity field. The largest of these perturbations are produced by the Earth's oblateness. Beyond the oblateness, there exist much smaller undulations in the gravity field. These variations produce much smaller, but certainly observable, effects on low-Earth orbiters.

The modeling of the Earth's gravity field using spherical harmonics is convenient for both numerical integration of satellite trajectories as well as analytic developments for the orbital perturbations. Both of these tasks are briefly reviewed. The equations of motion of a satellite moving in the gravitational field are derived. Computer implementation of these equations is facilitated by recurrence relations for the Associated Legendre Functions which are also given.

The principal perturbations produced on a satellite orbit are then defined and approximate expressions quantifying the perturbations are found using Kaula's approach.

2 Mathematical Model

2.1 Gravitational Potential

The common approach for modeling the gravitational field of a planetary body is through the spherical harmonic representation,

$$V = \frac{\mu}{r} \sum_{\ell=0}^{\infty} \left(\frac{a_e}{r} \right)^{\ell} \sum_{m=0}^{\ell} P_{\ell,m}(\sin\phi) \left[C_{\ell,m} \cos m\lambda + S_{\ell,m} \sin m\lambda \right] \tag{1}$$

where: μ is the product of the universal constant of gravitation G and the mass of the Earth M; a_e is the semi-major axis of the Earth's reference ellipsoid; r, ϕ, λ are the satellite distance, latitude, and longitude, respectively, in a body-fixed coordinate system; $C_{\ell m}$, $S_{\ell m}$ are spherical harmonic coefficients of degree ℓ and order m; and $P_{\ell m}$ are the Associated Legendre Functions of degree ℓ and order m. A gravitational model consists of a set of constants that specify μ, a_e and the $C_{\ell,m}$, $S_{\ell,m}$ coefficients. It should also be noted that such a set of constants also implicitly defines a body-fixed coordinate system. The coordinate system defined is precisely that which was used in the solution of the spherical harmonic coefficients.

This representation of the geopotential can be thought of as consisting of three constituent parts,

$$V = V_0 + V_1 + V_2 \tag{2}$$

The first part is simply the leading term of the expansion corresponding to the degree and order zero term. The Associated Legendre Function, P_{00} has a value of one as does the C_{00} coefficient. So the leading term is simply,

$$V_0 = \frac{\mu}{r} \tag{3}$$

This is the familiar potential resulting from treating the body as point mass and that used for deriving the fundamental results of two body motion.

The second part of the spherical harmonic representation are those terms (besides the above two body term) which do not have a longitude dependence. These are the terms corresponding to $m = 0$ and are denoted as the zonal contribution to the potential,

$$V_1 = \frac{\mu}{r} \sum_{\ell=1}^{\infty} \left(\frac{a_e}{r}\right)^{\ell} P_{\ell,0}(\sin\phi) C_{\ell,0} \tag{4}$$

The degree 2 zonal term models the contribution due to the planetary oblateness. As such, it is the second largest contributor to the overall potential following the central body contribution. (The degree 1 term is zero assuming that the center of the Earth-fixed coordinate system coincides with the center of mass of the Earth.) The notation J_ℓ is often used for the zonal coefficients instead of the above $C_{\ell,0}$. The two notations simply differ in sign,

$$J_\ell = -C_{\ell,0} \tag{5}$$

so that the zonal part of the potential could also be written in the form,

$$V_1 = -\frac{\mu}{r} \sum_{\ell=1}^{\infty} \left(\frac{a_e}{r}\right)^{\ell} P_{\ell,0}(\sin\phi) J_\ell \tag{6}$$

The C notation will be used throughout this paper.

The remaining part of the spherical harmonic representation is that part depending on longitude,

$$V_2 = \frac{\mu}{r} \sum_{\ell=1}^{\infty} \left(\frac{a_e}{r}\right)^{\ell} \sum_{m=1}^{\ell} P_{\ell,m}(\sin\phi) \left[C_{\ell,m} \cos m\lambda + S_{\ell,m} \sin m\lambda\right] \tag{7}$$

The largest longitudinal contributor to the potential is usually the degree 2 and order 2 terms. These terms represent the amount that the planet is “out of round” about the equator. (As with the degree 1 zonal coefficient, the degree 1 and order 1 coefficients will be zero under the assumption that the center of the coordinate system coincides with the center of mass.)

The spherical harmonic representation of the potential (Eq. 1) can then be written as,

$$\begin{aligned} V &= \frac{\mu}{r} \\ &+ \frac{\mu}{r} \sum_{\ell=1}^{\infty} \left(\frac{a_e}{r}\right)^{\ell} P_{\ell,0}(\sin\phi) C_{\ell,0} \\ &+ \frac{\mu}{r} \sum_{\ell=1}^{\infty} \left(\frac{a_e}{r}\right)^{\ell} \sum_{m=1}^{\ell} P_{\ell,m}(\sin\phi) \left[C_{\ell,m} \cos m\lambda + S_{\ell,m} \sin m\lambda\right] \end{aligned} \tag{8}$$

In the general case, where temporal variations in the potential exist (e.g., tides), the spherical harmonic representation is still valid though the geopotential coefficients ($C_{\ell,m}$, $S_{\ell,m}$) then become time dependent.

2.1.1 Spherical Harmonics

To better understand the utility of using a spherical harmonic representation of the geopotential a closer look at the spherical harmonic functions is required. The spherical harmonic functions are those functions formed by the product of the Associated Legendre Functions with $\cos m\lambda$ and $\sin m\lambda$ which appear in Eq. 1,

$$A_{\ell,m}(\phi,\lambda) = P_{\ell,m}(\sin\phi)\cos m\lambda \quad \text{and} \quad B_{\ell,m}(\phi,\lambda) = P_{\ell,m}(\sin\phi)\sin m\lambda \tag{9}$$

These functions are orthogonal. Thus, each function (for a given degree and order) can be thought of as contributing independent information with an amplitude given by their respective $C_{\ell,m}$ and $S_{\ell,m}$ coefficients.

In addition to being orthogonal, the qualitative shapes of the spherical harmonics are easily visualized. The zonal harmonics (corresponding to $m = 0$) have no longitude dependence and have ℓ zeroes between ± 90 degrees in latitude. So the even degree zonals are symmetric about the equator and the odd zonal are asymmetric. Note also that as the degree increases the number of zeroes in latitude increases and the harmonics represent finer and finer latitudinal variations in the potential. If only large scale (such as the oblateness) variations need to be modeled then only the lowest degree zonal terms need to be used.

The non-zonal harmonics all have longitudinal variations. The presence of the $\cos m\lambda$ and $\sin m\lambda$ give the functions $2m$ zeroes in longitude. And the Associated Legendre Functions have $\ell - m$ zeroes in latitude. So, similar to the zonals, the higher degree and order harmonics represent finer and finer spatial detail of the gravitational potential. The non-zonal coefficients are called tesserals and for the specific case of $\ell = m$ they are referred to as sectorials.

Generally, the spherical harmonics can be thought of as representing variations in the gravitational potential that have wavelengths of the circumference of the Earth divided by m in longitude, and divided by $\ell - m$ in latitude.

2.1.2 Normalization

The spherical harmonic coefficients appearing in Eq. 1 are unnormalized. These coefficients tend to very small values as the degree increases. This is partly a consequence of the nature of the Earth's gravity field but is for the most part due to the fact that the Associated Legendre Functions tend to large values as degree increases. Thus it is numerically advantageous to normalize the Associated Legendre Functions and the coefficients. The normalization is achieved by multiplying the Legendre functions by a scale factor depending on the degree and order of the function. Denoting normalized values by an overbar, the normalized Associated Legendre Functions are,

$$\bar{P}_{\ell,m} = \left[(2-\delta_{m0})(2\ell+1)\frac{(\ell-m)!}{(\ell+m)!}\right]^{1/2} P_{\ell,m} \tag{10}$$

where the Kronecker delta, δ_{m0}, is equal to 1 if m is zero and equal to 0 if m is greater than zero. The geopotential coefficients, $C_{\ell,m}$ and $S_{\ell,m}$, are normalized by the inverse of this scale factor,

$$\left\{ \begin{array}{c} \bar{C}_{\ell,m} \\ \bar{S}_{\ell,m} \end{array} \right\} = \left[\frac{1}{(2-\delta_{m0})(2\ell+1)}\frac{(\ell+m)!}{(\ell-m)!}\right]^{1/2} \left\{ \begin{array}{c} C_{\ell,m} \\ S_{\ell,m} \end{array} \right\} \tag{11}$$

The spherical harmonic expansion of the geopotential (Eq. 1) can now be written in terms of normalized quantities,

$$V = \frac{\mu}{r}\sum_{\ell=0}^{\infty}\left(\frac{a_e}{r}\right)^{\ell}\sum_{m=0}^{\ell}\bar{P}_{\ell,m}(\sin\phi)\left[\bar{C}_{\ell,m}\cos m\lambda + \bar{S}_{\ell,m}\sin m\lambda\right] \tag{12}$$

This is usually the preferred formulation for numerical implementations of the spherical harmonic representation. For many analytical developments (such as the effect on orbital motion) it is easier to work with the unnormalized form (Eq. 1).

2.1.3 Associated Legendre Functions

Evaluation of the spherical harmonic expansion requires evaluating the Associated Legendre Functions. This evaluation is most conveniently performed using recurrence relations. If only a few terms are needed (low degree and order) then explicit coding of the functions may be more desirable. In general the Associated Legendre Function of degree ℓ and order m is,

$$P_{\ell,m}(x) = \frac{(1-x^2)^{m/2}}{2^{\ell}}\sum_{k=0}^{\frac{\ell-m}{2}}(-1)^k\frac{(2\ell-2k)!}{k!(\ell-k)!(\ell-m-2k)!}x^{\ell-m-2k} \tag{13}$$

The first few of these functions are given in Table 1.

Table 1. Associated Legendre Functions $P_{\ell,m}(x)$

	Order m			
Degree ℓ	0	1	2	3
0	1			
1	x	$(1-x^2)^{1/2}$		
2	$\frac{1}{2}(3x^2-1)$	$3x(1-x^2)^{1/2}$	$3(1-x^2)$	
3	$\frac{1}{2}(5x^3-3x)$	$\frac{3}{2}(5x^2-1)(1-x^2)^{1/2}$	$15x(1-x^2)$	$15(1-x^2)^{3/2}$

Recurrence relations for evaluating these functions are generally in one of two forms. The difference is due to whether the recurrence is done holding the degree fixed or holding the order fixed. Either approach allows the computation of all the needed Legendre functions. In both cases, the starting values for the recurrences are the $\ell = m$ and $\ell = m + 1$ functions which are easily computed from,

$$P_{\ell,\ell}(x) = \frac{(2\ell-1)!}{2^{\ell-1}(\ell-1)!}(1-x^2)^{\ell/2} \tag{14}$$

$$P_{\ell,\ell-1}(x) = \frac{x}{(1-x^2)^{1/2}} P_{\ell,\ell}(x) \tag{15}$$

One recursion is to compute $P_{\ell,\ell}$ and $P_{\ell,\ell-1}$ and then compute the functions for all lower orders of degree ℓ using,

$$P_{\ell,m}(x) = \frac{1}{(\ell-m)(\ell+m+1)} \left[2(m+1)\frac{x}{(1-x^2)^{1/2}} P_{\ell,m+1}(x) - P_{\ell,m+2}(x) \right] \tag{16}$$

The alternate recursion is to compute $P_{\ell,\ell}$ and $P_{\ell+1,\ell}$ and then compute the functions for all higher degrees using,

$$P_{\ell,m}(x) = \frac{1}{\ell-m} \left[(2\ell-1)xP_{\ell-1,m}(x) - (\ell+m-1)P_{\ell-2,m}(x) \right] \tag{17}$$

The recurrence relations can also be rewritten to directly work with the normalized Associated Legendre Functions. The recurrence equivalent to Eq. 16 in normalized form is,

$$\bar{P}^*_{\ell,m}(x) = 2(m+1)\left[\tfrac{1}{(\ell+m+1)(\ell-m)}\right]^{1/2} \frac{x}{(1-x^2)^{1/2}} \bar{P}^*_{\ell,m+1}(x) - \left[\tfrac{(\ell+m+2)(\ell-m-1)}{(\ell+m+1)(\ell-m)}\right]^{1/2} \bar{P}^*_{\ell,m+2}(x) \tag{18}$$

where $\bar{P}_{\ell,m} = \bar{P}^*_{\ell,m}$ if $m > 0$ and $\bar{P}_{\ell,0} = \frac{1}{\sqrt{2}}\bar{P}^*_{\ell,0}$ when $m = 0$.

The recurrence equivalent to Eq. 17 in normalized form is,

$$\bar{P}_{\ell,m}(x) = \left[\tfrac{(2\ell-1)(2\ell+1)}{(\ell-m)(\ell+m)}\right]^{1/2} x\bar{P}_{\ell-1,m}(x) - \left[\tfrac{(2\ell+1)(\ell+m-1)(\ell-m-1)}{(2\ell-3)(\ell+m)(\ell-m)}\right]^{1/2} \bar{P}_{\ell-2,m}(x) \tag{19}$$

2.2 Gravitational Acceleration

The gravitational acceleration at any given location is obtained by computing the gradient of the potential. Since the potential is given as a function of Earth-fixed spherical coordinates, it is most convenient to compute the gradient in the same system. In Earth-fixed spherical coordinates, this gradient is,

$$\vec{a} = \nabla V = \frac{\partial V}{\partial r}\vec{u}_r + \frac{1}{r}\frac{\partial V}{\partial \phi}\vec{u}_\phi + \frac{1}{r\cos\phi}\frac{\partial V}{\partial \lambda}\vec{u}_\lambda \tag{20}$$

where $\vec{u}_r$, $\vec{u}_\phi$ and $\vec{u}_\lambda$ are unit vectors in the r, ϕ, λ basis. This basis has $\vec{u}_r$ pointing along the radius vector to the satellite, $\vec{u}_\phi$ is in the direction of increasing north latitude and $\vec{u}_\lambda$ is in the direction of increasing East longitude. The acceleration vector obtained from this expression will be the inertial acceleration for the point of interest. Though, as noted, the components of the acceleration are given in the Earth-fixed coordinate system. For most applications it will be desired to have the components of the acceleration expressed in an inertial (non-rotating) coordinate system. This is accomplished by applying the appropriate coordinate transformation from the spherical coordinates to the desired coordinate system. So as a first step, the components of the inertial acceleration in the Earth-fixed (rotating) coordinate system are obtained. Substituting for the gravitational potential (Eq. 1) and taking the indicated partials in Eq. 20 gives the acceleration vector,

$$\begin{aligned}\vec{a} &= \left\{-\frac{\mu}{r^2}\sum_{\ell=0}^{\infty}(\ell+1)\left(\frac{a_e}{r}\right)^{\ell}\sum_{m=0}^{\ell}P_{\ell,m}(\sin\phi)\left[C_{\ell,m}\cos m\lambda + S_{\ell,m}\sin m\lambda\right]\right\}\vec{u}_r \\ &+ \left\{\frac{\mu}{r^2}\sum_{\ell=1}^{\infty}\left(\frac{a_e}{r}\right)^{\ell}\sum_{m=0}^{\ell}\frac{\partial P_{\ell,m}(\sin\phi)}{\partial\phi}\left[C_{\ell,m}\cos m\lambda + S_{\ell,m}\sin m\lambda\right]\right\}\vec{u}_\phi \\ &+ \left\{\frac{\mu}{r^2}\sum_{\ell=1}^{\infty}\left(\frac{a_e}{r}\right)^{\ell}\sum_{m=1}^{\ell}m\frac{P_{\ell,m}(\sin\phi)}{\cos\phi}\left[-C_{\ell,m}\sin m\lambda + S_{\ell,m}\cos m\lambda\right]\right\}\vec{u}_\lambda \end{aligned} \tag{21}$$

Notice that the leading term of the radial component (degree and order equal to zero) is simply the expected two-body gravitational acceleration $-\mu/r^2$. Also, if only zonal terms are used ($m = 0$), then the longitudinal component of the acceleration is zero.

Next, the Earth-fixed Cartesian components of the acceleration can be obtained by rotating from the spherical coordinates to the x, y, z basis. Let the components of the acceleration in spherical coordinates be represented by,

$$\vec{a} = a_r\vec{u}_r + a_\phi\vec{u}_\phi + a_\lambda\vec{u}_\lambda \tag{22}$$

where the components a_r, a_ϕ and a_λ are given in Eq. 21. The acceleration vector in Cartesian coordinates can be written as,

$$\vec{a}_{xyz} = a_x\vec{u}_x + a_y\vec{u}_y + a_z\vec{u}_z \tag{23}$$

where $\vec{u}_x$, $\vec{u}_y$ and $\vec{u}_z$ are the Cartesian unit vectors in the Earth-fixed (rotating) coordinate system. The Cartesian components of the acceleration can be obtained from the spherical coordinate components through the standard transformation,

$$\begin{vmatrix} a_x \\ a_y \\ a_z \end{vmatrix} = \begin{vmatrix} \cos\phi\cos\lambda & -\sin\phi\cos\lambda & -\sin\lambda \\ \cos\phi\sin\lambda & -\sin\phi\sin\lambda & \cos\lambda \\ \sin\phi & \cos\phi & 0 \end{vmatrix} \begin{vmatrix} a_r \\ a_\phi \\ a_\lambda \end{vmatrix} \tag{24}$$

Having obtained the Earth-fixed Cartesian components of the acceleration one further coordinate transformation is necessary to obtain the acceleration components in the defined inertial coordinate system. If the matrix T represents the coordinate transformation from the Earth-fixed system to the inertial coordinate system, then the acceleration components in the inertial system will be,

$$\vec{a}_{XYZ} = T\vec{a}_{xyz} \tag{25}$$

where $\vec{a}_{XYZ}$ is the inertial acceleration vector in inertial coordinates,

$$\vec{a}_{XYZ} = a_X\vec{u}_X + a_Y\vec{u}_Y + a_Z\vec{u}_Z \tag{26}$$

with $\vec{u}_X$, $\vec{u}_Y$ and $\vec{u}_Z$ being the unit vectors of the Cartesian inertial coordinate system. In component form, this final transformation will have the structure,

$$\begin{vmatrix} a_X \\ a_Y \\ a_Z \end{vmatrix} = \begin{vmatrix} T_{11} & T_{12} & T_{13} \\ T_{21} & T_{22} & T_{23} \\ T_{31} & T_{32} & T_{33} \end{vmatrix} \begin{vmatrix} a_x \\ a_y \\ a_z \end{vmatrix} \tag{27}$$

The actual elements of the transformation matrix T depend on the inertial coordinate system being used. In the most general case, this transformation will account for polar motion (the motion of the spin axis with respect to the Earth crust), Earth rotation (the largest effect) and, precession and nutation (the motion of the spin axis with respect to the stars). In the simplest case, all of these effects are neglected except for Earth rotation. This defines a coordinate system with the same z axis as the Earth-fixed system but not rotating with the Earth. For many applications such a system is effectively inertial. The transformation from the Earth-fixed system to this non-rotating system is simply,

$$T = \begin{vmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{vmatrix} \tag{28}$$

where θ is the Greenwich Hour Angle (the angle from a reference direction, usually the Vernal Equinox, to the Greenwich meridian).

2.2.1 Example Equations of Motion

The equations of motion for a satellite moving in a gravitational field modeled in spherical harmonics are given by Eq. 27. In the most general case, the components of the acceleration in spherical coordinates are evaluated (Eq. 21) complete to the highest degree and order of the gravity model being used. These components are then rotated to Earth-fixed Cartesian components (Eq. 24) and finally to inertial coordinates as shown in Eq. 27.

As an example, the resulting equations of motion due to the second and third degree zonal will be presented. The second degree zonal is the most important (largest) term of the potential and is necessary for the most rudimentary modeling of orbital motion. The third degree zonal is responsible for long period (on the order of 100 days) variations in the elements of an orbiting spacecraft and should be included for any long term orbit propagation studies. Assume that the inertial coordinate system to be used is one neglecting polar motion, precession and nutation. That is, the matrix defined in Eq. 28 is the transformation from Earth-fixed (rotating) coordinates to the inertial (non-rotating) system. Letting, X, Y, Z represent the location of the satellite in this inertial system, the equations of motion for the satellite are,

$$\begin{aligned} \ddot{X} = \; & -\frac{\mu}{r^2}\frac{X}{r} \quad -\frac{\mu}{r^2}\left(\frac{a_e}{r}\right)^2 C_{2,0}\left[\frac{15}{2}\left(\frac{Z}{r}\right)^2 - \frac{3}{2}\right]\frac{X}{r} \\ & -\frac{\mu}{r^2}\left(\frac{a_e}{r}\right)^3 C_{3,0}\left[\frac{35}{2}\left(\frac{Z}{r}\right)^3 - \frac{15}{2}\frac{Z}{r}\right]\frac{X}{r} \end{aligned} \tag{29}$$

$$\begin{aligned} \ddot{Y} = \; & -\frac{\mu}{r^2}\frac{Y}{r} \quad -\frac{\mu}{r^2}\left(\frac{a_e}{r}\right)^2 C_{2,0}\left[\frac{15}{2}\left(\frac{Z}{r}\right)^2 - \frac{3}{2}\right]\frac{Y}{r} \\ & -\frac{\mu}{r^2}\left(\frac{a_e}{r}\right)^3 C_{3,0}\left[\frac{35}{2}\left(\frac{Z}{r}\right)^3 - \frac{15}{2}\frac{Z}{r}\right]\frac{Y}{r} \end{aligned} \tag{30}$$

$$\begin{aligned} \ddot{Z} = \; & -\frac{\mu}{r^2}\frac{Z}{r} \quad -\frac{\mu}{r^2}\left(\frac{a_e}{r}\right)^2 C_{2,0}\left[\frac{15}{2}\left(\frac{Z}{r}\right)^2 - \frac{9}{2}\right]\frac{Z}{r} \\ & -\frac{\mu}{r^2}\left(\frac{a_e}{r}\right)^3 C_{3,0}\left[\frac{35}{2}\left(\frac{Z}{r}\right)^4 - 15\left(\frac{Z}{r}\right)^2 + \frac{3}{2}\right] \end{aligned} \tag{31}$$

The Greenwich Hour Angle, θ, appearing in the transformation matrix (Eq. 28) does not appear in these equations of motion, because there is not any longitudinal variation in the potential when only zonal terms are used. So the acceleration of the spacecraft is independent of it's longitudinal location with respect to Greenwich.

3 Earth's Gravity Field

Numerous spherical harmonic models of the Earth's gravity field have been developed. These models are primarily based on the Earth-based tracking of low-Earth orbit satellites. Other data types that are valuable in estimating the Earth's gravity field include surface gravity measurements, satellite-to-satellite tracking and more recently, satellite radar-altimeter measurements of the ocean surface. The maximum degree (ℓ) of the spherical harmonic representations of the Earth is more than 300 in some models. Models based solely on satellite tracking data usually have a maximum degree of approximately 50. Indeed, for most satellite applications, high degree models are not needed due to the insensitivity of the satellite motions to the small scale features represented in such models.

A good general purpose model for satellite applications is the JGM-2 model developed jointly by the NASA Goddard Space Flight Center (GSFC) and the University of Texas Center for Space Research (*Nerem et al.*, 1994). This model is based on the tracking data of Earth satellites and surface gravity measurements; the model is complete to degree and order 70. The accuracy of this model is greatly improved with respect to earlier Goddard Earth Models (GEM). In addition to estimating the spherical harmonic coefficients of the gravity field, estimates of the accuracy of those coefficients are also computed. Such accuracy estimates are very valuable when attempting to estimate the orbit error which may be induced when using the model for orbit propagation.

Recently, a newer JGM (JGM-3) model has been produced (*Tapley et al.*, 1996). This model is based on more satellite tracking data and includes some GPS tracking of the Topex/Poseidon spacecraft. The overall accuracy is improved with respect to JGM-2.

Table 2. The JGM-2 Gravity Model.
Normalized Zonals $\bar{C}_{\ell 0}$ in units of 1×10^{-9}

ℓ	m	$\bar{C}_{\ell m}$	σ	ℓ	m	$\bar{C}_{\ell m}$	σ	ℓ	m	$\bar{C}_{\ell m}$	σ
2	0	−484165.48	.11	25	0	4.62	1.11	48	0	4.52	1.27
3	0	957.12	.03	26	0	8.21	1.29	49	0	−.16	1.28
4	0	540.14	.26	27	0	4.09	1.23	50	0	−3.88	1.27
5	0	68.46	.16	28	0	−12.47	1.33	51	0	−6.22	1.25
6	0	−150.00	.35	29	0	−3.50	1.31	52	0	.92	1.23
7	0	90.95	.36	30	0	9.41	1.39	53	0	5.65	1.21
8	0	49.30	.52	31	0	5.01	1.39	54	0	.95	1.18
9	0	26.70	.57	32	0	−4.63	1.32	55	0	2.26	1.18
10	0	53.89	.66	33	0	−.45	1.47	56	0	−2.85	1.15
11	0	−49.31	.75	34	0	−6.43	1.30	57	0	−2.34	1.13
12	0	35.61	.83	35	0	5.32	1.38	58	0	−3.81	1.12
13	0	39.13	.98	36	0	−6.05	1.27	59	0	−.17	1.09
14	0	−22.00	1.05	37	0	−5.36	1.34	60	0	−3.05	1.08
15	0	4.56	1.13	38	0	.47	1.23	61	0	.58	1.05
16	0	−5.55	1.09	39	0	2.06	1.42	62	0	1.63	1.04
17	0	17.74	1.04	40	0	−1.76	1.30	63	0	−2.60	1.02
18	0	6.66	.99	41	0	−1.86	1.38	64	0	−2.23	1.00
19	0	−2.47	.95	42	0	1.66	1.36	65	0	.06	.97
20	0	19.79	1.13	43	0	5.46	1.38	66	0	−.49	.94
21	0	7.72	1.01	44	0	2.15	1.32	67	0	.13	.91
22	0	−10.81	1.28	45	0	−5.47	1.32	68	0	.83	.87
23	0	−22.48	1.08	46	0	−1.46	1.27	69	0	1.60	.81
24	0	−2.84	1.26	47	0	−.85	1.33	70	0	−.81	.78

The JGM-2 normalized zonal coefficients, complete to degree 70, are given in Table 2. The nonzonal coefficients, up to degree and order 30, are given in Table 3 (the model contains coefficients up to degree and order 70). Along with the coefficient values in each table, the estimated uncertainty (σ) of the individual coefficients is also given. The gravitational constant and equatorial radius specified for the JGM-2 model are,

$$\mu = GM = 3.986004415 \times 10^{14}\ \mathrm{m}^3/\mathrm{s}^2 \qquad \text{and} \qquad a_e = 6378136.3\ \mathrm{m} \tag{32}$$

Several points can be made by examining the coefficient values and their uncertainties. Foremost, the value of the second degree zonal coefficient is seen to be more than two orders of magnitude larger than any other coefficient. The next largest coefficients are those of degree 2 and order 2. Analogous to the second degree zonal which represents the oblateness of the Earth, these coefficients correspond to the ellipticity about the equator.

Also evident is that the magnitude of the coefficients decreases significantly as the degree increases (keep in mind that these are normalized coefficients and effectively

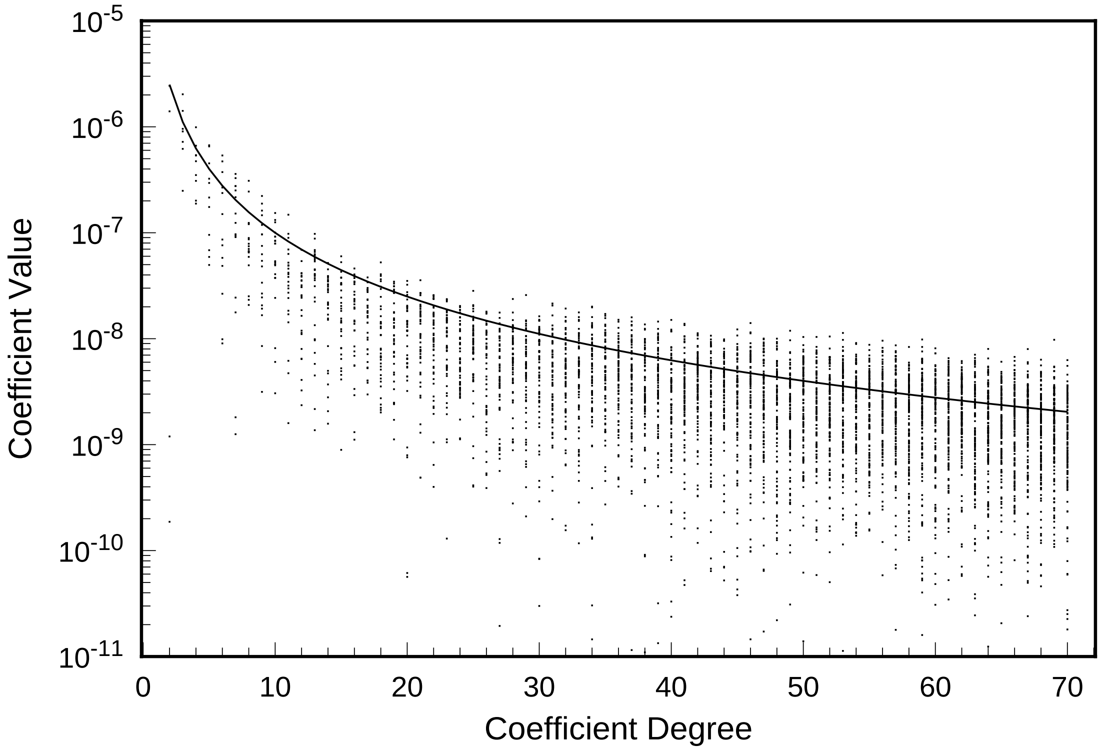

Figure 1: Absolute value of the normalized JGM-2 coefficients and Kaula's Rule.

have equal weight in their total contribution to the gravitational potential). This characteristic has been formalized in the so-called "Kaula's Rule" (*Kaula*, 1966). This rule gives the expected size of the Earth's normalized harmonic coefficients of degree ℓ to be $\pm 10^{-5}/\ell^2$. This rule of thumb allows one to estimate the expected magnitude of a gravitational coefficient if a value is not otherwise known (this is particularly convenient for higher degree coefficients for which accurate estimates have generally not been obtained). The magnitude of the JGM-2 coefficients (complete to degree and order 70) are plotted in Figure 1 along with Kaula's Rule.

Table 3. The JGM-2 Gravity Model. Normalized Sectorials and Tesserals $\bar{C}_{\ell m}$ and $\bar{S}_{\ell m}$ in units of 1×10^{-9}

ℓ	m	$\bar{C}_{\ell m}$	σ	$\bar{S}_{\ell m}$	σ	ℓ	m	$\bar{C}_{\ell m}$	σ	$\bar{S}_{\ell m}$	σ
2	1	−.19	.00	1.20	.00	2	2	2439.08	.12	−1400.11	.12
3	1	2028.40	.42	248.81	.42	3	2	904.41	.27	−619.23	.29
4	1	−536.37	.24	−473.42	.23	4	2	350.35	.43	662.87	.44
5	1	−59.12	.84	−95.53	.85	5	2	653.39	.68	−323.78	.71
6	1	−76.13	.49	26.56	.49	6	2	48.65	.80	−373.79	.84
7	1	275.83	1.20	96.78	1.18	7	2	327.88	1.24	94.03	1.27
8	1	23.28	.90	59.20	.89	8	2	78.76	1.14	66.25	1.21
9	1	146.27	1.35	20.65	1.27	9	2	24.53	1.60	−33.78	1.59
10	1	83.63	1.15	−131.96	1.09	10	2	−91.97	1.29	−52.51	1.40
11	1	14.28	1.42	−27.17	1.28	11	2	17.00	1.63	−97.75	1.60
12	1	−54.00	1.29	−41.26	1.21	12	2	11.91	1.29	31.93	1.43
13	1	−53.66	1.39	40.07	1.29	13	2	55.82	1.63	−62.76	1.58
14	1	−19.40	1.37	27.48	1.35	14	2	−35.87	1.27	−3.70	1.43
15	1	11.75	1.32	7.10	1.35	15	2	−20.35	1.59	−32.88	1.55
16	1	27.79	1.32	34.39	1.35	16	2	−22.40	1.41	25.99	1.55
17	1	−27.53	1.39	−29.04	1.53	17	2	−18.25	1.37	9.79	1.45
18	1	3.94	1.27	−39.12	1.32	18	2	12.56	1.46	13.98	1.59
19	1	−9.04	1.40	−1.12	1.58	19	2	31.30	1.42	−3.94	1.54
20	1	8.49	1.26	5.83	1.38	20	2	20.49	1.46	13.84	1.58
21	1	−18.17	1.49	27.02	1.63	21	2	−2.78	1.45	4.60	1.50
22	1	13.50	1.26	−1.95	1.47	22	2	−23.98	1.48	−.64	1.57
23	1	8.24	1.48	15.78	1.64	23	2	−14.72	1.55	−5.20	1.64
24	1	−2.76	1.31	−5.16	1.53	24	2	−1.15	1.57	14.73	1.64
25	1	4.62	1.44	−11.10	1.61	25	2	19.19	1.61	7.86	1.77
26	1	−1.93	1.34	−6.94	1.56	26	2	−.52	1.59	13.53	1.67
27	1	4.09	1.52	−.56	1.68	27	2	5.54	1.57	.81	1.70
28	1	−6.20	1.40	6.01	1.59	28	2	−14.99	1.54	−10.13	1.65
29	1	.62	1.56	−8.35	1.73	29	2	−5.24	1.58	−2.98	1.69
30	1	−.86	1.45	−.46	1.63	30	2	−9.00	1.56	.40	1.67
3	3	721.15	.20	1414.04	.20						
4	3	990.26	.23	−201.01	.22	4	4	−188.49	.21	308.85	.21
5	3	−451.90	.39	−215.10	.38	5	4	−295.08	.25	49.67	.24
6	3	57.95	.65	9.03	.62	6	4	−86.30	.37	−471.67	.37
7	3	250.90	.83	−216.63	.84	7	4	−275.55	.55	−123.86	.53
8	3	−20.81	1.15	−86.66	1.09	8	4	−244.84	.79	70.29	.79
9	3	−161.92	1.32	−75.14	1.33	9	4	−8.53	1.01	19.21	.99
10	3	−6.04	1.48	−153.61	1.40	10	4	−84.16	1.26	−78.91	1.26
11	3	−29.83	1.56	−148.04	1.52	11	4	−40.62	1.43	−63.08	1.41
12	3	38.51	1.54	24.34	1.46	12	4	−68.96	1.50	4.08	1.52
13	3	−22.39	1.54	97.62	1.47	13	4	−1.37	1.56	−13.41	1.57
14	3	35.04	1.51	21.32	1.43	14	4	1.58	1.45	−20.86	1.49
15	3	52.74	1.57	14.22	1.45	15	4	−42.80	1.48	8.17	1.52
16	3	−34.04	1.45	−23.74	1.33	16	4	40.44	1.45	46.00	1.47
17	3	7.15	1.62	7.96	1.47	17	4	7.91	1.43	23.22	1.48
18	3	−4.21	1.56	−2.74	1.43	18	4	52.54	1.43	2.01	1.41
19	3	−9.92	1.62	−2.49	1.48	19	4	15.12	1.50	−5.79	1.52
20	3	−5.85	1.61	35.11	1.53	20	4	5.44	1.49	−23.45	1.47
21	3	19.98	1.80	20.95	1.64	21	4	−5.95	1.48	16.95	1.47
22	3	10.15	1.68	12.55	1.62	22	4	−5.30	1.52	17.65	1.54
23	3	−23.21	1.79	−18.37	1.70	23	4	−23.64	1.57	7.78	1.58
24	3	−3.91	1.73	−8.97	1.60	24	4	7.60	1.68	3.33	1.69
25	3	−11.37	1.77	−16.15	1.69	25	4	9.30	1.69	−.01	1.73
26	3	9.93	1.75	2.26	1.65	26	4	18.01	1.74	−17.54	1.79
27	3	4.37	1.88	6.63	1.75	27	4	.94	1.68	9.60	1.70
28	3	2.61	1.82	12.69	1.73	28	4	−.28	1.72	3.82	1.80
29	3	.65	1.90	−10.54	1.79	29	4	−25.82	1.66	.70	1.72
30	3	−.81	1.82	−14.75	1.72	30	4	−2.50	1.73	−.98	1.81
5	5	174.97	.25	−669.65	.25						
6	5	−267.19	.20	−536.52	.20	6	6	9.89	.24	−237.09	.25
7	5	1.81	.36	17.72	.37	7	6	−359.04	.16	151.77	.16
8	5	−25.15	.45	89.25	.45	8	6	−65.16	.35	309.24	.35

Table 3. Continued.

ℓ	m	$\bar{C}_{\ell m}$	σ	$\bar{S}_{\ell m}$	σ	ℓ	m	$\bar{C}_{\ell m}$	σ	$\bar{S}_{\ell m}$	σ
9	5	−16.66	.72	−54.31	.73	9	6	62.68	.46	222.43	.47
10	5	−49.30	.91	−50.64	.91	10	6	−37.42	.59	−79.02	.58
11	5	38.30	1.18	48.89	1.16	11	6	−1.60	.85	34.47	.88
12	5	30.69	1.33	8.00	1.32	12	6	3.25	1.03	39.23	1.00
13	5	57.85	1.50	65.93	1.46	13	6	−35.67	1.23	−5.78	1.27
14	5	29.45	1.54	−16.88	1.52	14	6	−19.38	1.33	2.80	1.28
15	5	14.20	1.55	8.11	1.51	15	6	33.62	1.41	−37.54	1.47
16	5	−14.51	1.47	−1.31	1.40	16	6	14.84	1.41	−34.65	1.38
17	5	−17.53	1.50	5.34	1.40	17	6	−14.19	1.36	−28.20	1.40
18	5	7.25	1.53	24.82	1.49	18	6	13.88	1.45	−14.41	1.42
19	5	12.66	1.62	26.97	1.46	19	6	−1.72	1.37	17.54	1.41
20	5	−11.96	1.62	−6.51	1.54	20	6	12.91	1.57	−.94	1.49
21	5	1.30	1.75	.49	1.61	21	6	−14.56	1.46	.49	1.48
22	5	−2.29	1.72	1.05	1.65	22	6	9.33	1.54	−8.42	1.49
23	5	1.05	1.82	−1.12	1.71	23	6	−14.29	1.53	15.59	1.51
24	5	−6.54	1.74	−20.37	1.66	24	6	3.71	1.65	2.96	1.61
25	5	−9.05	1.82	−4.54	1.71	25	6	18.14	1.65	.41	1.64
26	5	9.00	1.78	8.47	1.73	26	6	11.56	1.74	−9.97	1.70
27	5	15.71	1.87	13.78	1.79	27	6	1.13	1.70	7.63	1.70
28	5	10.16	1.87	−2.62	1.81	28	6	−3.08	1.75	9.45	1.71
29	5	−7.11	1.90	1.43	1.84	29	6	12.57	1.71	4.56	1.72
30	5	−3.62	1.88	−5.36	1.81	30	6	.03	1.77	2.60	1.75
7	7	1.25	.31	24.43	.31						
8	7	67.16	.20	74.63	.21	8	8	−123.89	.38	120.46	.39
9	7	−118.49	.38	−96.59	.37	9	8	188.43	.28	−3.15	.28
10	7	8.14	.35	−3.06	.36	10	8	40.64	.38	−91.76	.39
11	7	4.72	.63	−89.51	.62	11	8	−6.18	.43	24.20	.42
12	7	−18.61	.66	35.21	.66	12	8	−25.73	.51	16.44	.52
13	7	2.17	.98	−7.35	.98	13	8	−9.89	.74	−9.66	.72
14	7	37.46	1.02	−4.71	1.02	14	8	−34.69	.82	−15.08	.82
15	7	59.97	1.23	6.44	1.23	15	8	−32.45	1.04	21.79	1.01
16	7	−7.59	1.22	−8.46	1.22	16	8	−21.12	1.03	5.61	1.01
17	7	23.48	1.29	−6.02	1.29	17	8	37.89	1.19	4.02	1.14
18	7	6.70	1.29	5.84	1.31	18	8	30.99	1.23	2.23	1.21
19	7	7.41	1.41	−7.38	1.39	19	8	31.23	1.29	−10.80	1.24
20	7	−20.06	1.44	.06	1.46	20	8	5.34	1.27	4.02	1.30
21	7	−10.16	1.53	4.79	1.50	21	8	−18.11	1.39	3.52	1.35
22	7	14.92	1.64	4.16	1.63	22	8	−25.64	1.38	2.59	1.41
23	7	−5.60	1.67	−1.94	1.67	23	8	5.06	1.41	−2.23	1.41
24	7	−4.49	1.70	3.46	1.66	24	8	15.98	1.52	−5.08	1.55
25	7	8.10	1.70	−6.39	1.70	25	8	6.47	1.58	1.84	1.58
26	7	−2.32	1.76	2.88	1.73	26	8	4.12	1.63	2.04	1.65
27	7	−12.43	1.75	−2.65	1.74	27	8	−10.48	1.67	−10.53	1.67
28	7	−1.05	1.82	5.84	1.82	28	8	−3.81	1.67	−4.83	1.70
29	7	−4.17	1.83	−5.28	1.82	29	8	−14.62	1.70	10.61	1.68
30	7	5.99	1.83	−.08	1.84	30	8	1.59	1.71	2.78	1.72
9	9	−48.12	.48	96.60	.48						
10	9	125.45	.20	−37.55	.20	10	10	100.29	.38	−24.27	.38
11	9	−31.68	.42	41.97	.42	11	10	−52.22	.32	−18.37	.32
12	9	41.63	.32	25.25	.32	12	10	−6.43	.36	30.65	.36
13	9	24.50	.51	45.15	.53	13	10	40.85	.40	−37.30	.40
14	9	32.68	.49	28.68	.47	14	10	38.80	.43	−2.07	.43
15	9	12.31	.79	37.79	.80	15	10	10.60	.57	14.77	.57
16	9	−23.17	.78	−39.09	.77	16	10	−11.99	.59	11.94	.59
17	9	2.98	.93	−28.98	.93	17	10	−4.03	.78	17.80	.78
18	9	−18.99	.99	36.39	.97	18	10	5.11	.82	−5.46	.82
19	9	3.34	1.10	6.74	1.10	19	10	−33.16	.99	−7.41	.98
20	9	18.34	1.18	−6.46	1.17	20	10	−32.31	1.04	−5.89	1.05
21	9	16.25	1.23	9.23	1.22	21	10	−11.08	1.14	−1.56	1.13
22	9	6.25	1.30	9.78	1.27	22	10	5.44	1.20	24.04	1.22
23	9	.13	1.44	−14.92	1.43	23	10	15.02	1.23	−2.88	1.22
24	9	−10.24	1.46	−18.65	1.42	24	10	9.93	1.26	17.70	1.30

Table 3. Continued.

ℓ	m	$\bar{C}_{\ell m}$	σ	$\bar{S}_{\ell m}$	σ	ℓ	m	$\bar{C}_{\ell m}$	σ	$\bar{S}_{\ell m}$	σ
25	9	−28.29	1.56	20.30	1.54	25	10	10.03	1.35	−3.96	1.35
26	9	−11.49	1.56	2.06	1.53	26	10	−13.83	1.41	−4.40	1.45
27	9	.92	1.64	11.66	1.62	27	10	−13.96	1.49	−.02	1.51
28	9	9.23	1.69	−8.99	1.66	28	10	−9.75	1.48	8.87	1.50
29	9	−2.22	1.72	−1.64	1.70	29	10	13.85	1.59	4.47	1.61
30	9	−9.56	1.76	−11.32	1.74	30	10	3.00	1.52	−6.44	1.56
11	11	45.82	.57	−69.53	.57						
12	11	11.35	.15	−6.47	.15	12	12	−2.36	.27	−11.04	.27
13	11	−44.49	.44	−4.50	.44	13	12	−31.30	.18	88.05	.18
14	11	15.50	.23	−39.01	.23	14	12	8.51	.23	−30.99	.23
15	11	−.89	.45	18.28	.45	15	12	−32.56	.27	15.50	.28
16	11	19.28	.32	−2.93	.32	16	12	19.73	.24	6.89	.25
17	11	−15.87	.58	10.24	.57	17	12	28.96	.37	20.40	.37
18	11	−7.86	.54	2.12	.54	18	12	−29.46	.33	−16.31	.33
19	11	15.57	.75	10.71	.76	19	12	−2.47	.55	8.87	.54
20	11	14.67	.77	−19.15	.78	20	12	−6.42	.53	18.20	.53
21	11	7.78	.95	−35.64	.97	21	12	−2.90	.76	15.11	.75
22	11	−4.67	1.01	−17.13	1.02	22	12	2.85	.76	−7.92	.75
23	11	8.58	1.13	16.64	1.15	23	12	16.70	.97	−12.70	.95
24	11	13.79	1.20	18.71	1.22	24	12	11.76	.99	−5.33	.96
25	11	2.45	1.30	9.48	1.31	25	12	−8.81	1.12	12.46	1.11
26	11	−2.06	1.29	.86	1.31	26	12	−17.22	1.17	1.42	1.11
27	11	3.96	1.36	−8.58	1.41	27	12	−7.76	1.25	.74	1.24
28	11	−4.61	1.42	.89	1.43	28	12	1.12	1.24	11.68	1.19
29	11	−7.40	1.45	7.24	1.49	29	12	−2.63	1.33	−2.95	1.32
30	11	−9.76	1.51	9.48	1.54	30	12	15.10	1.31	−11.48	1.28
13	13	−61.46	.28	68.68	.27						
14	13	32.16	.06	45.17	.06	14	14	−51.88	.24	−4.97	.23
15	13	−28.55	.24	−4.13	.24	15	14	5.26	.09	−24.43	.09
16	13	13.80	.09	1.11	.09	16	14	−19.20	.18	−38.78	.18
17	13	16.41	.24	20.53	.24	17	14	−14.13	.11	11.38	.11
18	13	−6.33	.11	−34.87	.11	18	14	−8.06	.19	−13.02	.18
19	13	−7.53	.29	−28.09	.29	19	14	−4.58	.14	−13.14	.14
20	13	27.50	.18	6.99	.18	20	14	11.80	.22	−14.39	.22
21	13	−19.11	.34	13.84	.35	21	14	20.25	.23	7.56	.23
22	13	−17.07	.30	19.62	.30	22	14	11.17	.33	7.82	.33
23	13	−11.43	.50	−4.44	.51	23	14	7.64	.40	−2.47	.40
24	13	−3.07	.56	3.39	.56	24	14	−19.82	.52	−1.73	.52
25	13	7.49	.73	−12.00	.74	25	14	−20.74	.67	7.30	.68
26	13	.54	.84	1.33	.84	26	14	8.34	.80	7.06	.79
27	13	−4.26	.98	−3.80	.99	27	14	17.64	.95	10.12	.97
28	13	1.06	1.03	7.23	1.05	28	14	−8.00	1.04	−12.32	1.03
29	13	−1.11	1.15	−2.00	1.16	29	14	−5.73	1.11	−3.24	1.11
30	13	14.86	1.12	1.81	1.14	30	14	5.22	1.13	6.85	1.13
15	15	−19.24	.37	−4.94	.37						
16	15	−14.52	.12	−32.70	.12	16	16	−37.32	.54	3.35	.54
17	15	5.31	.27	5.27	.27	17	16	−30.14	.26	3.84	.25
18	15	−40.58	.17	−20.22	.17	18	16	10.79	.44	6.84	.43
19	15	−17.82	.30	−14.11	.30	19	16	−21.58	.33	−6.83	.33
20	15	−25.93	.24	−.76	.25	20	16	−11.98	.46	.06	.45
21	15	17.62	.34	10.59	.34	21	16	7.87	.43	−6.54	.43
22	15	25.77	.39	4.80	.40	22	16	.40	.52	−6.97	.51
23	15	18.45	.50	−3.57	.49	23	16	6.34	.58	11.49	.59
24	15	6.38	.60	−15.85	.61	24	16	9.16	.69	3.21	.68
25	15	−4.30	.68	−7.20	.68	25	16	.97	.82	−12.98	.82
26	15	−13.74	.81	8.23	.82	26	16	1.63	.88	−5.85	.88
27	15	−2.13	.97	2.18	.97	27	16	3.62	.93	3.43	.92
28	15	−11.69	1.02	−2.50	1.04	28	16	−4.06	1.04	−13.15	1.05
29	15	−9.09	1.16	−7.47	1.16	29	16	−.40	1.19	−14.90	1.19
30	15	−.29	1.19	−.08	1.22	30	16	−9.40	1.21	4.49	1.21
17	17	−34.08	.63	−19.62	.64						
18	17	3.54	.26	4.53	.26	18	18	2.39	.71	−10.87	.71

Table 3. Continued.

ℓ	m	$\bar{C}_{\ell m}$	σ	$\bar{S}_{\ell m}$	σ	ℓ	m	$\bar{C}_{\ell m}$	σ	$\bar{S}_{\ell m}$	σ
19	17	29.10	.52	−15.01	.52	19	18	34.55	.31	−9.52	.31
20	17	4.39	.36	−13.52	.36	20	18	14.74	.58	−.79	.59
21	17	−6.77	.53	−7.18	.53	21	18	25.72	.48	−10.97	.48
22	17	8.81	.45	−14.35	.45	22	18	9.96	.62	−16.22	.62
23	17	−5.53	.52	−12.51	.52	23	18	8.34	.64	−14.20	.64
24	17	−12.34	.60	−5.86	.60	24	18	−1.12	.66	−10.01	.67
25	17	−14.96	.72	−3.20	.72	25	18	.74	.76	−15.39	.77
26	17	−11.94	.83	8.25	.83	26	18	−13.63	.87	5.19	.87
27	17	3.46	.98	.89	.97	27	18	−2.96	1.00	8.85	1.01
28	17	13.07	1.03	−5.12	1.03	28	18	5.32	1.04	−4.20	1.05
29	17	−1.05	1.15	−3.98	1.15	29	18	−4.74	1.07	−4.33	1.10
30	17	−6.35	1.22	−4.28	1.21	30	18	−11.94	1.17	−7.66	1.17
19	19	−2.41	.77	4.80	.77						
20	19	−3.21	.44	10.71	.44	20	20	4.41	.85	−11.92	.86
21	19	−26.98	.65	16.65	.65	21	20	−26.74	.40	16.19	.40
22	19	13.84	.56	−3.76	.56	22	20	−16.61	.71	20.08	.72
23	19	−5.37	.68	10.73	.67	23	20	8.18	.60	−5.34	.60
24	19	−4.27	.67	−8.11	.67	24	20	−4.91	.73	8.84	.73
25	19	7.84	.66	9.81	.66	25	20	−7.30	.78	−.40	.79
26	19	−2.13	.76	3.33	.76	26	20	6.60	.80	−11.54	.80
27	19	−.13	.84	−2.84	.84	27	20	−.89	.87	3.75	.87
28	19	5.99	.99	23.70	.99	28	20	−1.43	1.00	7.21	1.01
29	19	−5.61	1.17	5.77	1.17	29	20	−7.16	1.05	4.76	1.06
30	19	−12.95	1.20	2.29	1.19	30	20	−4.31	1.17	12.25	1.16
21	21	8.32	.84	−3.86	.85						
22	21	−25.03	.59	23.66	.58	22	22	−9.65	.96	2.22	.96
23	21	15.38	.72	11.82	.73	23	22	−17.71	.47	4.66	.47
24	21	6.03	.74	13.52	.74	24	22	3.68	.81	−3.86	.80
25	21	10.70	.75	7.46	.76	25	22	−13.34	.68	4.11	.68
26	21	−8.98	.84	2.01	.84	26	22	10.87	.80	7.16	.80
27	21	4.94	.77	−6.79	.77	27	22	−5.72	.87	3.58	.86
28	21	6.56	.86	6.41	.86	28	22	−1.77	.89	−6.63	.90
29	21	−9.51	.90	−5.28	.91	29	22	12.29	.93	−1.00	.94
30	21	−10.92	1.07	−6.70	1.07	30	22	−4.53	1.05	−8.72	1.05
23	23	3.06	.91	−10.49	.91						
24	23	−6.38	.67	−8.10	.67	24	24	11.93	1.02	−3.73	1.02
25	23	8.20	.76	−11.85	.75	25	24	3.99	.49	−8.33	.49
26	23	1.23	.84	11.25	.84	26	24	8.03	.84	14.53	.83
27	23	−5.06	.78	−10.12	.77	27	24	−.12	.66	−2.23	.66
28	23	5.60	.91	3.14	.91	28	24	10.60	.78	−13.45	.78
29	23	−2.61	.81	2.79	.81	29	24	.21	.82	−2.57	.82
30	23	4.79	.88	−9.64	.88	30	24	−2.57	.91	−2.83	.90
25	25	10.77	.88	4.33	.87						
26	25	3.72	.69	−.69	.69	26	26	.39	.97	1.95	.97
27	25	12.04	.66	5.30	.65	27	26	−6.56	.42	−2.23	.42
28	25	6.39	.85	−17.67	.85	28	26	11.77	.65	3.89	.65
29	25	6.03	.68	8.42	.68	29	26	7.96	.56	−6.84	.56
30	25	3.15	.91	−16.28	.90	30	26	1.47	.84	12.22	.83
27	27	8.00	.81	1.02	.81						
28	27	−8.10	.73	1.07	.73	28	28	7.12	.88	7.15	.88
29	27	−7.55	.58	−.89	.58	29	28	9.83	.37	−5.87	.37
30	27	−7.46	.76	12.54	.77	30	28	−5.35	.64	−7.61	.64
29	29	13.25	.96	−5.41	.98						
30	29	4.01	.69	2.04	.70	30	30	3.08	1.16	7.60	1.15

The estimated accuracy of the various coefficients shown in Tables 2 and 3 show that the lower degree coefficients are the best determined and the accuracy degrades as the degree increases. This variation in accuracy is a reflection of the fact that satellite tracking data was used to solve for the coefficients. As discussed in later sections, the sensitivity of the satellites to the harmonic coefficients decreases as the degree increases. That is, the low degree coefficients produce large perturbations to the orbital motion and the high degree coefficients produce much smaller perturbations. The ability to recover high degree coefficients is a direct function of the accuracy of the tracking data being utilized and also the geographic distribution of that data. Since the high degree coefficients represent fine scale features in the gravity field it is necessary to have wide geographic coverage, and accurate tracking data, to completely capture such details. The low degree coefficients on the other hand represent the large scale features (continental in size) and it is possible to accurately model such details given sparser geographic coverage.

Overall, the accuracy of the geopotential acceleration is dependent on the constants μ, $\bar{C}_{\ell,m}$ and $\bar{S}_{\ell,m}$ and the accuracy of the coordinate transformation from body-fixed to inertial coordinates. The accuracy is also clearly dependent on the degree of truncation of the infinite series describing the potential. It should be noted that the quantity a_e enters the potential strictly as a scaling factor and thus does not affect the accuracy of the geopotential computation. Of these possible error sources, the accuracy of the $\bar{C}_{\ell,m}$ and $\bar{S}_{\ell,m}$ coefficients is currently the limiting factor in precise low-Earth orbit determination.

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